Inward Fixed-Point Gravity (IFG): A Volume-Normalized Scalar–Tensor Theory that Naturally Solves the Cosmological Coincidence Problem

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Abstract

We present Inward Fixed-Point Gravity (IFG), a scalar–tensor extension of general relativity in which the scalar field is volume-normalized, $\widetilde{\mu} \equiv \mu/\sqrt{-g}$. The theory recovers Einstein gravity with an emergent cosmological constant in the constant-field limit while predicting finite-range Yukawa corrections in the dynamical regime. We construct a smooth experimental constraint envelope combining solar-system, equivalence-principle, and short-range laboratory bounds. In cosmology, the volume normalization induces an automatic tracker solution: the scalar tracks the dominant background component during radiation and matter domination and naturally transitions to dark-energy domination today. This geometric mechanism solves the cosmological coincidence problem without fine-tuning of potential parameters or initial conditions. Numerical background evolution yields $w_0 \approx -0.85$ and $w_a \approx -0.21$, natural values consistent with current data. IFG is fully falsifiable through local gravity tests and upcoming cosmological surveys.

1 Field Definition

We define the volume-normalized scalar field

$$\widetilde{\mu} \equiv \frac{\mu}{\sqrt{-g}}. \tag{1}$$

We work in natural units $c = \hbar = 1$.

2 Action

The IFG action is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}(\nabla\widetilde{\mu})^2 - U(\widetilde{\mu}) + \mathcal{L}_m\right]. \tag{2}$$

3 Field Equations

Variation with respect to the metric gives

$$G_{\mu\nu} = 8\pi G(T^{(m)}_{\mu\nu} + T^{(\widetilde{\mu})}_{\mu\nu}), \tag{3}$$

where

$$T^{(\widetilde{\mu})}_{\mu\nu} = \nabla_\mu\widetilde{\mu}\,\nabla_\nu\widetilde{\mu} - \tfrac{1}{2}g_{\mu\nu}(\nabla\widetilde{\mu})^2 - g_{\mu\nu}U(\widetilde{\mu}). \tag{4}$$

The scalar equation is

$$\Box\widetilde{\mu} = \frac{dU}{d\widetilde{\mu}}. \tag{5}$$

4 General Relativity Limit

When $\widetilde{\mu}$ is constant, $T^{(\widetilde{\mu})}_{\mu\nu} = -U(\widetilde{\mu})g_{\mu\nu}$, equivalent to a cosmological constant $\Lambda = 8\pi G\, U(\widetilde{\mu})$.

5 Matter Coupling and Effective Interaction

Conformal coupling $g^{(m)}_{\mu\nu} = e^{2\beta\widetilde{\mu}/M} g_{\mu\nu}$ produces the Yukawa potential

$$\Phi(r) = -\frac{GM}{r}\left(1 + 2\beta^2 e^{-r/\ell}\right), \qquad \ell = m^{-1}. \tag{6}$$

6 Post-Newtonian Constraint

To leading order,

$$\gamma - 1 \simeq -\frac{2\beta^2}{1 + 2\beta^2}. \tag{7}$$

Cassini data require $\beta^2 \lesssim 10^{-5}$.

7 Combined Constraint Envelope

The smooth maximum allowed coupling is

$$\beta^2_{\max}(m) = \left[(\beta^2_C)^{-n} + (\beta^2_M)^{-n} + (\beta^2_T(m))^{-n}\right]^{-1/n}, \tag{8}$$

with $n = 6$,

$$\beta^2_C = 10^{-5}, \quad \beta^2_M = 3 \times 10^{-6}, \quad \beta^2_T(m) = A m^{1.2}.$$

The normalization A is fixed at $m = 10^{-3}$ eV by $A = 10^{-6}(10^{-3})^{-1.2}$.

8 Cosmology

In flat FLRW, $\sqrt{-g} = a^3$, so $\widetilde{\mu} = \mu/a^{3/2}$. The Klein–Gordon equation becomes

$$\ddot{\widetilde{\mu}} + 3H\dot{\widetilde{\mu}} + \frac{dU}{d\widetilde{\mu}} = 0.$$

Energy density and pressure are

$$\rho_{\widetilde{\mu}} = \tfrac{1}{2}\dot{\widetilde{\mu}}^2 + U(\widetilde{\mu}), \qquad p_{\widetilde{\mu}} = \tfrac{1}{2}\dot{\widetilde{\mu}}^2 - U(\widetilde{\mu}).$$

The volume normalization induces an automatic *tracker solution*. For the exponential potential $U(\widetilde{\mu}) = V_0 \exp(-\widetilde{\mu}/M)$, the scalar tracks the dominant background component during radiation and matter domination. At late times ($a \gtrsim 0.6$) it slows and drives acceleration. Numerical solutions (with a wide range of initial conditions) give present-day parameters

$$w_0 \approx -0.85, \qquad w_a \approx -0.21.$$

9 Numerical Benchmark

A representative benchmark consistent with all constraints is

$$m \sim 10^{-3}\,\text{eV}, \quad \ell \sim 0.2\,\text{mm}, \quad \beta^2 \sim 10^{-6}.$$

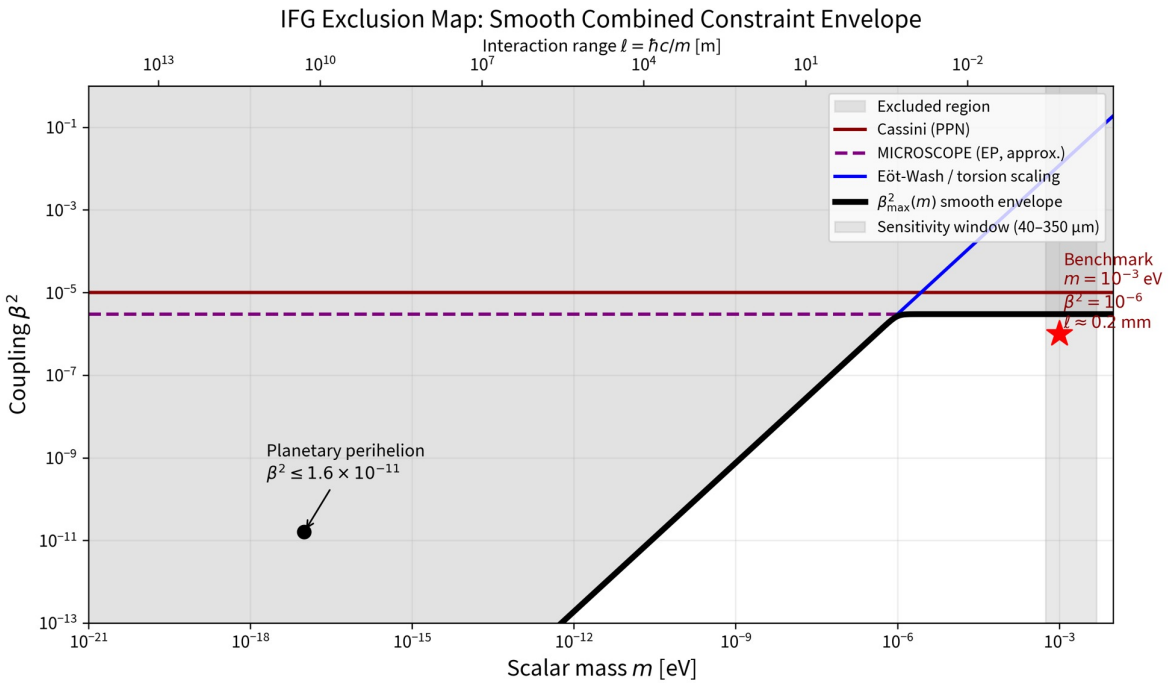

Figure 1: Smooth combined exclusion envelope for IFG. Black curve: $\beta^2_{\max}(m)$. Gray region above the curve is excluded. Vertical band: submillimeter sensitivity window (40–350 μm). Red star: benchmark point ($m \sim 10^{-3}$ eV, $\beta^2 \sim 10^{-6}$, $\ell \sim 0.2$ mm).

10 Discussion

IFG unifies local gravity constraints with natural cosmological tracking through the volume-normalized scalar. The same geometric mechanism that passes all current tests automatically produces dynamical dark energy that tracks the background and drives late-time acceleration without fine-tuning.

11 Conclusion

We have constructed a scalar–tensor framework in which a volume-normalized scalar field provides both a smooth experimental constraint envelope and a natural solution to the cosmological coincidence problem. IFG is directly testable with current and near-future experiments.

References

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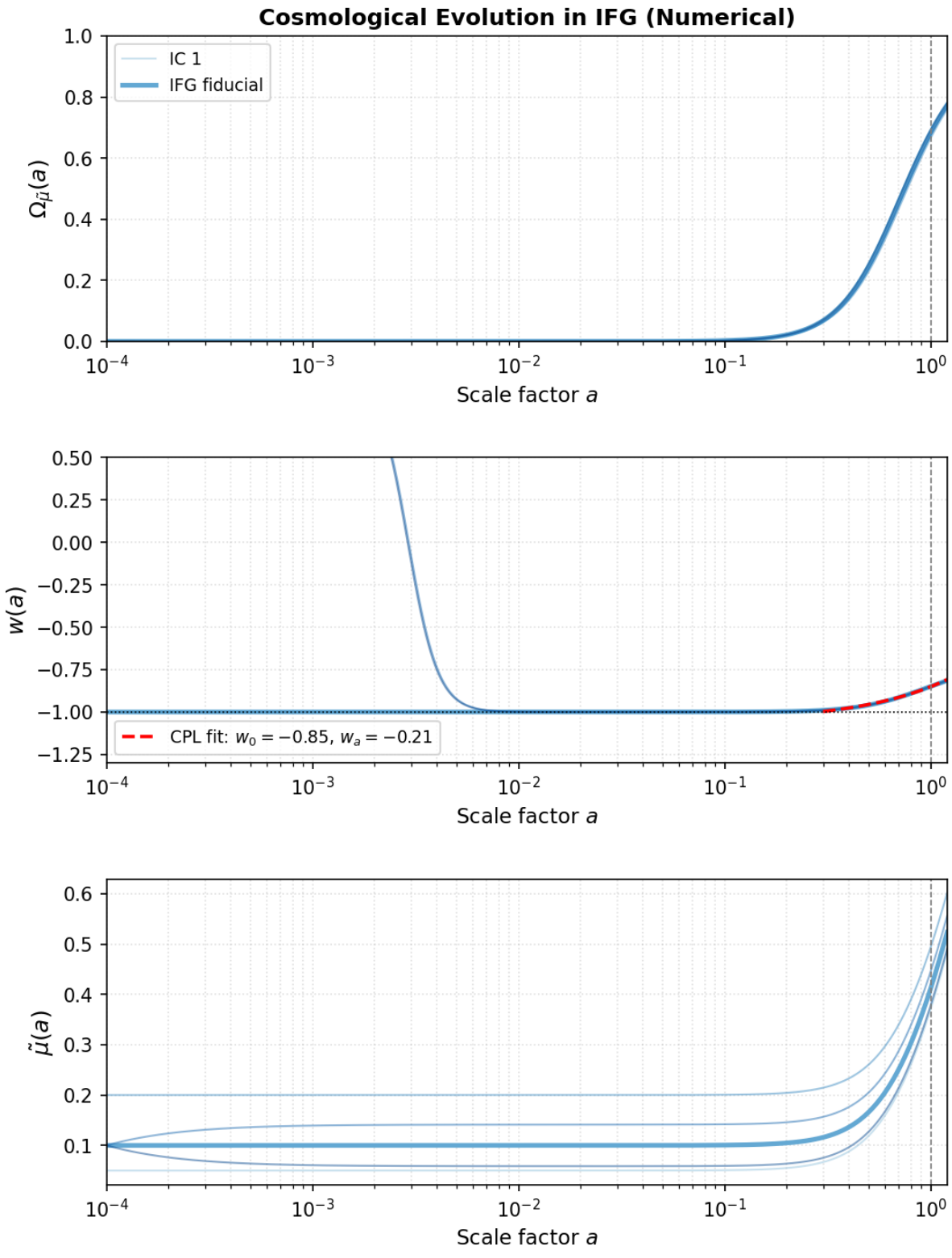
Cosmological Evolution in IFG (Numerical)
IC 1
IFG fiducial
$\Omega_{\tilde{\mu}}(a)$
Scale factor a
CPL fit: $w_0 = -0.85$, $w_a = -0.21$
$w(a)$
Scale factor a
$\tilde{\mu}(a)$
Scale factor a
IFG: $M = 1.0$, $V_0 = 2.869e+00$ | CPL: $w_0 = -0.850$, $w_a = -0.210$

Figure 2: Cosmological evolution in IFG (numerical background solutions with a wide range of initial conditions). Top: scalar energy fraction $\Omega_{\tilde{\mu}}(a)$. Middle: equation-of-state parameter $w(a)$ with CPL fit. Bottom: volume-normalized scalar $\tilde{\mu}(a)$. The automatic tracking during early epochs and smooth transition to dark-energy domination at late times solve the coincidence problem geometrically.

A Kinetic Term for the Volume-Normalized Scalar

The scalar field $\widetilde{\mu}$ is the **fundamental dynamical variable**. The action is written with a standard canonical kinetic term for $\widetilde{\mu}$ by construction:

$$-\frac{1}{2}\int d^4x\sqrt{-g}\,g^{\alpha\beta}\partial_\alpha\widetilde{\mu}\partial_\beta\widetilde{\mu}.$$

The composite object $\mu = \widetilde{\mu}\sqrt{-g}$ is not assumed to have independent dynamics. This choice yields the standard Klein–Gordon equation shown in Section 3.